Cambridge Futures Modelling

Measuring the impacts of the housing and transport infrastructure gaps on employment growth in the Greater Cambridge city region

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Non-technical Summary

This study is commissioned by Cambridge Ahead to update the understanding of whether, and if so when, the housing and transport infrastructure pressures currently faced by the Greater Cambridge city region[1] may become so high that a tipping point is triggered and the economy (as measured by employment levels) begins to go into decline, thus resulting a large potential loss for the UK as a whole.

This analysis is carried out through building on the modelling that was undertaken for and extensively used by the Cambridgeshire and Peterborough Independent Economic Review (CPIER, 2018). The modelling work was carried out by the Cambridgeshire and Peterborough Futures Modelling Team[2]. This team's work builds on a series of Cambridge Futures studies since 1997 (See Cambridge Futures, 1999) and their extensive model databases, analyses and policy studies over a period of over fifty years in this and many other regions around the world. The study team is very aware of the fact that the team itself is a Cambridge research group, and has redoubled its effort to retain its neutral and independent perspective regarding the modelling and analysis.

The modelling work has:

(1) collected available economic, social, land use and transport data since CPIER (2018), thus building up a picture about "what has happened" since then, and since covid;

(2) collected available data about the business environment among the top international competitor regions of Cambridge, thus identifying of the likely alternative locations for tech innovation firms and institutions;

(3) updated the Cambridgeshire and Peterborough Futures model with the above information and used the model to estimate "what could happen next".

Headline findings

The findings from this study points towards a very serious challenge for the future development of the Greater Cambridge city region. The findings suggest that:

(1) In the event that no significant new policy actions were taken to address the housing and transport gaps over the next decade, employment in the city region is likely to reverse its growth trend and decline from 2031 onwards. This trend would then accelerate if the lack of policy action would continue. Once set in the declining trend, the housing and transport pressures would ease, but the loss of agglomeration economies relative to the city region's global competitors could then drive further falls in jobs, resulting in 124,900-143,600 fewer jobs by 2051 relative to the trend expected in the Joint Local Plan consultation. See

[1] This is defined as the local authority areas of Cambridge City and South Cambridgeshire.

[2] This team is led by Professor Ying Jin at the Martin Centre for Architectural and Urban Studies, Department of Architecture, University of Cambridge,

Table 1 and Figure 1-Figure 3;

(2) The growth trend in jobs as expected in the Joint Local Plan consultation is itself a modest one. Relative to what was envisaged by CPIER (2018) which could be regarded as a fair and level-headed assessment of the employment growth potential of the city region, the trend implied in the Joint Local Plan consultation is expecting 112,600 fewer jobs by 2051 than CPIER (2018). See

Table 1 and Figure 1-Figure 3);

(3) The resulting loss to the economy is very significant. We assume that under the housing and transport gap scenarios, all UK cities and regions suffer equally and the jobs lost are resulting from the loss of high skilled jobs to overseas competitors, and from the subsequent loss of local service jobs arising from the high skilled job losses. In 2021, the average GVA per job in the city region is £67,000. Assuming a discount rate of 3% per year, the net present value of the GVA loss from the jobs difference when discounted back to 2021 is between £79.3 to £93.7 bn for the period 2021-2051 relative the Joint Local Plan trend, and between £164.1 to £178.5 bn relative to the potential of jobs growth expected by CPIER (2018), see Table 2[3].

(4) The loss to the economy estimated above is only limited to the production output lost as a result of jobs growth coming well below expected. The wider ramifications of this loss will also be very significant, for example the social, cultural, and quality of life improvements that the expected jobs growth could bring. Such losses have not been included here.

[3] Note here we assume the average GVA per worker remains constant throughout the period to 2051 – this is a very conservative estimate of the net present values (NPVs); alternatively, one could see the estimates as NPVs estimated with a higher discount rate (i.e. the assumed 3% discount rate plus expected per worker productivity increase).

Table 1: Differences in workplace employment: a comparison among projections and model tests (thousands of working persons)

	1981	1991	2001	2011	2021	2031	2041	2051
Cambridge City								
Census workplace population	58.3	70.1	74.7	88.1	87.4			
Local Plan consultation (2023) expected growth					100.1	112.0	124.0	
Local Plan extrapolation to 2051								135.9
Impacted by housing & transport gaps (a)						84.1	81.2	73.2
Impacted by housing & transport gaps (b)						97.5	90.7	78.1
CPIER expectation (2018)					116.6	144.0	168.9	198.1
Trends of 2010-2015 continues					116.7	152.8	199.4	260.3
Difference: Impacted (a) - Local Plan						**-27.9**	**-42.8**	**-62.7**
Difference: Impacted (b) - Local Plan						**-14.5**	**-33.3**	**-57.8**
Difference: Local Plan - CPIER expectation						**-32.0**	**-44.9**	**-62.1**
South Cambs								
Census workplace population	31.8	47.0	62.7	72.5	80.4			
Local Plan consultation (2023) expected growth					93.4	114.3	135.2	
Local Plan extrapolation to 2051								156.1
Impacted by housing & transport gaps (a)						89.2	95.1	93.9
Impacted by housing & transport gaps (b)						95.5	89.3	70.3
CPIER expectation (2018)					101.2	130.9	164.4	206.5
Trends of 2010-2015 continues					101.2	139.9	203.1	294.9
Difference: Impacted (a) - Local Plan						**-25.1**	**-40.1**	**-62.2**
Difference: Impacted (b) - Local Plan						**-18.8**	**-45.9**	**-85.8**
Difference: Local Plan - CPIER expectation						**-16.6**	**-29.2**	**-50.4**
Greater Cambridge (City+S Cambs)								
Census workplace population	90.1	117.1	137.3	160.6	167.8			
Local Plan consultation (2023) expected growth					193.5	226.3	259.2	
Local Plan extrapolation to 2051								292.0
Impacted by housing & transport gaps (a)						173.3	176.3	167.1
Impacted by housing & transport gaps (b)						193.0	180.0	148.4
CPIER expectation (2018)					217.8	274.9	333.3	404.6
Trends of 2010-2015 continues					217.9	292.6	402.5	555.2
Difference: Impacted (a) - Local Plan						**-53.0**	**-82.9**	**-124.9**
Difference: Impacted (b) - Local Plan						**-33.3**	**-79.2**	**-143.6**
Difference: Local Plan - CPIER expectation						**-48.5**	**-74.1**	**-112.6**

Notes: 1. Workplace employment is measured in Census Workplace Population, net of full time students who are employed; 2. Because of covid pandemic, Census 2021 Workplace Population is deemed to underestimate workplace employment because of working from home under covid, etc; 3. 'Impacted by housing & transport gaps (a)' is modelled using the 2021 Workplace Population as the base year population, with current levels of hybrid work persisting over time; 4. 'Impacted by housing & transport gaps (b)' is modelled using a higher Workplace Population estimated from the Local Plan trends, with hybrid working first reducing to half of currently levels by 2031 and then increasing again from then on; 5. As Impact (a) represents a relatively low demand for housing and transport while Impact (b) a relatively high demand, the most likely outcome is expected to be between the two tests.

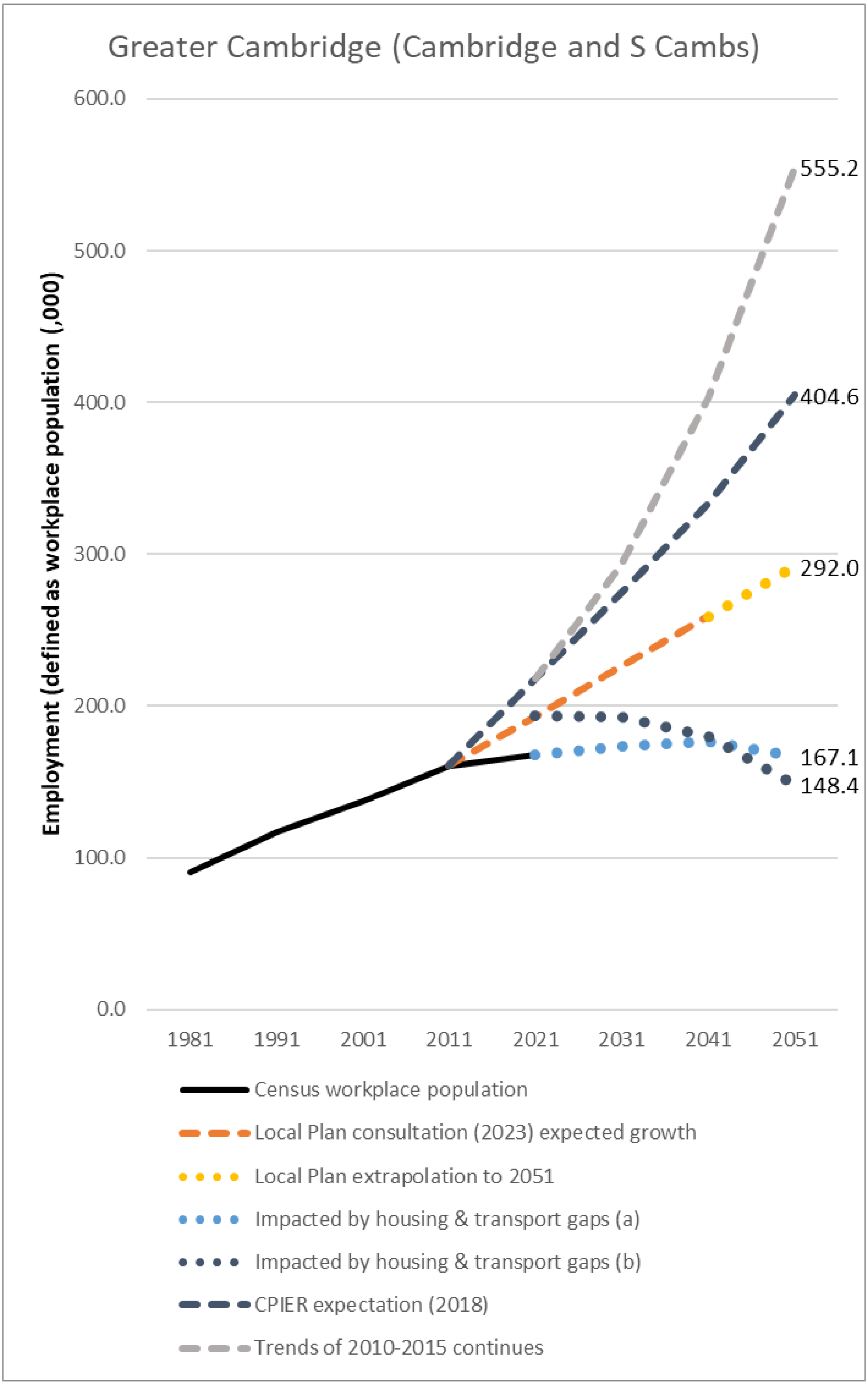

Figure 1: Impacts on employment as a result of housing and transport supply gaps: **Greater Cambridge city region**

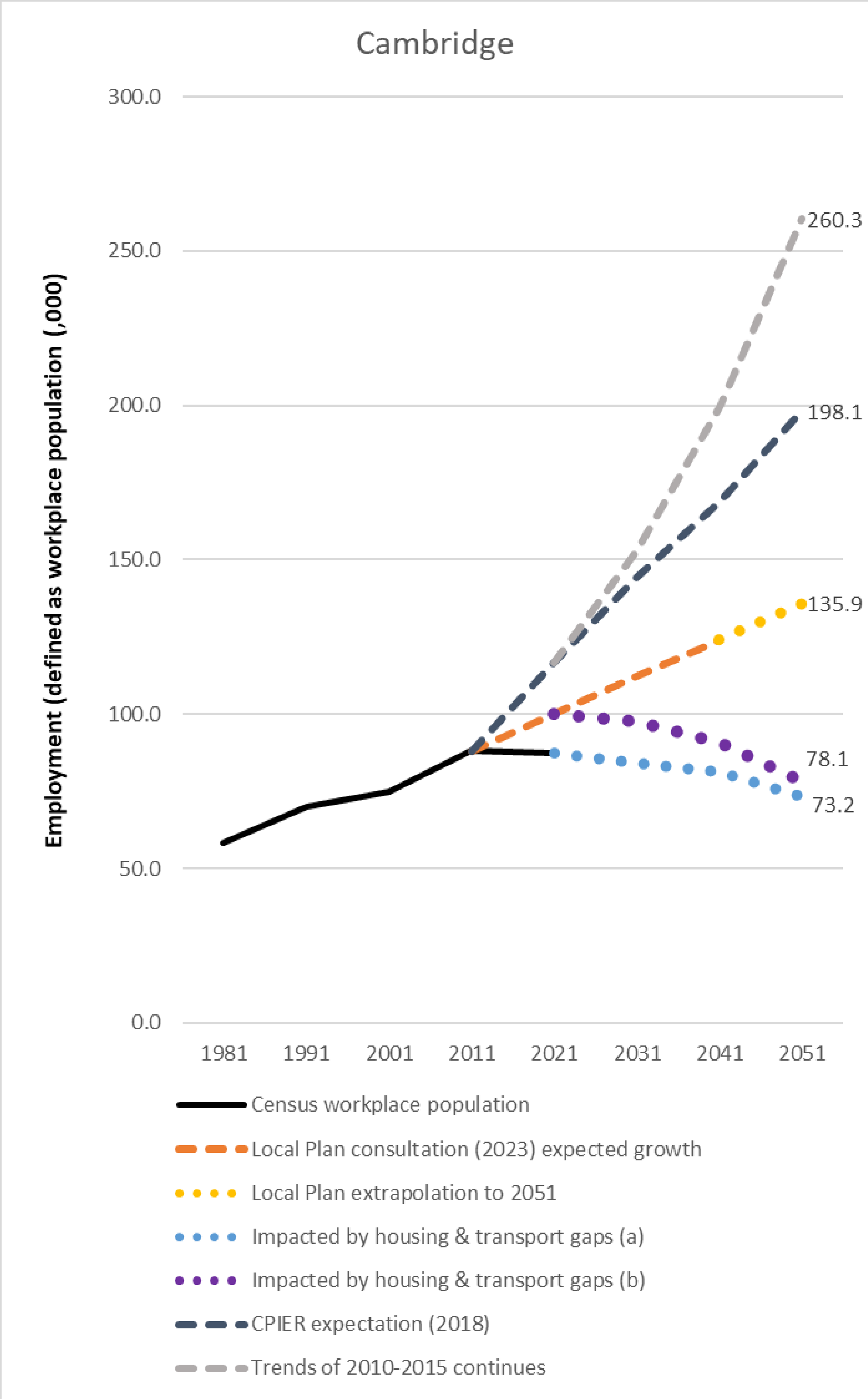

Figure 2: Impacts on employment as a result of housing and transport supply gaps: **Cambridge City**

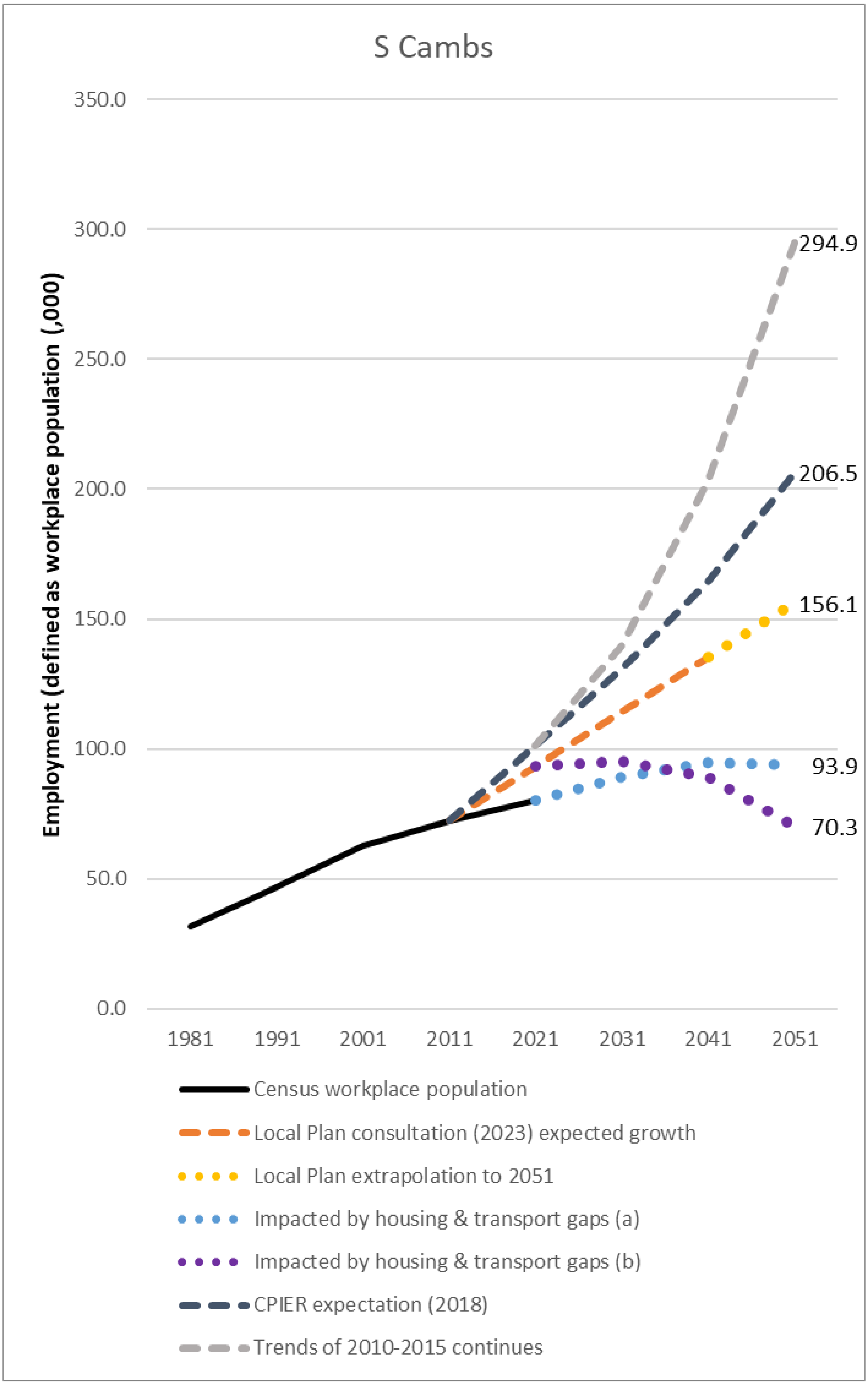

Figure 3: Impacts on employment as a result of housing and transport supply gaps: **South Cambridgeshire**

Table 2: Differences in workplace employment and loss in gross value added (GVA): a comparison among projections and scenarios

	Employment difference (000)			Estimated earnings loss (£bn 2021 prices)			
	2021-31	2031-41	2041-51	2021-31	2031-41	2041-51	**2021-51**
				Assuming constant salary no discounting			
Difference: Impacted (a) - Local Plan	-53.0	-82.9	-124.9	-35.5	-45.5	-69.6	**-150.7**
Difference: Impacted (b) - Local Plan	-33.3	-79.2	-143.6	-22.3	-37.7	-74.7	**-134.7**
Difference: Local Plan - CPIER expectation	-48.5	-74.1	-112.6	-32.5	-41.1	-62.5	**-136.1**
				Net present value with 3% discounting			
Difference: Impacted (a) - Local Plan				-31.4	-29.7	-32.5	**-93.7**
Difference: Impacted (b) - Local Plan				-19.8	-24.6	-34.9	**-79.3**
Difference: Local Plan - CPIER expectation				-28.8	-26.8	-29.2	**-84.8**

Note: An average annual GVA per job of £67,000 is assumed, as per ONS data for 2021.

The rest of this summary outlines the main assumptions and the reasoning behind the findings. For more information about the model theories and equations, see Appendix A, and for the detailed model assumptions, inputs and outputs, see Appendix B.

Main assumptions for the modelling

First of all, the modelling work is underpinned with a firm commitment to the highest standards of planning, design, construction and nature conservation. We assume that the city region will be able to harness economic growth in a way that improves rather than harms its environmental, social and cultural aspects, and based on this premise, growth in the economy and employment represents will enhance quality of life and climate change actions.

Secondly, the model assumptions regarding the gaps in housing and transport infrastructure are implemented through the following steps (a)-(d):

(a) The starting point of the model tests is from a simulation model representing the state of balances between the supply and demand in housing and transport in 2011. The assumption is that the state of the balances of 2011 (which was calibrated using 2011 observed datasets during the CPIER model development) represented a reasonable provision of housing and transport. This assumption is corroborated by the fact that at the time, the city region economy was able to compete reasonably strongly and attract investment at a global scale;

(b) On housing, the model collects input data for simulation runs every ten years: for all model zones for 2021, the Census 2021 dwellings data is used; for 2031 and 2041, the proposed housing growth numbers from the Joint Local Plan consultation are used for Cambridge City and South Cambridgeshire and the housing construction trend based inputs established in the CPIER work are used for all the other model zones including the rest of Cambridgeshire and Peterborough; for 2051, a straight-line trend projection from the Joint

Local Plan consultation are used for Cambridge City and South Cambridgeshire, and for the other model zones, the housing construction trend based inputs established in the CPIER work are used. In the past decade, housing growth in the city region has been phenomenal and the Joint Local Plan consultation also proposes substantial new growth. However, the city region's housing supply and affordability issues have arisen from (a) strong population and employment growth in the past two decades, particularly of highly skilled workers within Cambridge City and towards the southern parts of South Cambridgeshire; (b) slower delivery of housing in the wider commuting catchment beyond Greater Cambridge – CPIER (2018) estimates that for the city region's growth potential, 6000-8000 dwellings or even 9000 dwellings would be needed per year for Cambridgeshire and Peterborough Combined Authority area; (c) The majority of the largest on-going and new housing schemes are sited in suburbs and exurbs with inconvenient transport to the core of employment growth, which thus diminishes the impact of new housing supply. The model has been able to detect these issues through spatial modelling; (d) in the past decade, the rate of income growth lags well behind housing prices and rents;

(c) On transport, the model takes account of recent delivery of the A14 improvements (opened in 2020), Cambridge North Station, Cambridge South Station, urban transport improvements delivered under the City Deal and Devolution Deal (including greenways, cycle routes and park-and-ride expansions), 20-mile zones and walkable neighbourhoods, etc, for year 2021. For future years, it is assumed that two on-going transit schemes would be delivered by 2031 (i.e. Cambourne to Cambridge and Waterbeach to Cambridge) and other radial corridors would have delivered city-access transit schemes by 2041. No assumptions have been made regarding East-West Rail as it is not certain when it would be completed. The main driver of traffic congestion and poor connectivity is still peak time traffic, which has already reached the pre-covid levels on core working days of the week;

(d) Besides above, the CPIER model has been updated to include an updated 2021/2022 model year – because Covid has affected the Census 2021 datasets in various ways and also the home-work activity patterns, the data issues have been circumvented through modelling different test options (see below).

Thirdly, not all current gaps in infrastructure has been included in the model. For avoidance of doubt the model has not accounted for the impacts arising from pressures of water, energy supply or specific types of business space (e.g. specialist labs).

Why employment is a good indicator?

Given that the economic output per working person in the UK economy has been fairly stable in the past decade, and this stability is likely to remain little changed for some years yet (this is alternatively known as the 'productivity puzzle'), the impacts on economic growth can be directly gauged through that of employment, at least for the foreseeable future. Employment is therefore used in this study as a core and tangible measure of economic activity.

Employment, or 'jobs', is also a core social and cultural concept – the extent of employment growth or decline is expected to affect the social and cultural lives of the city region.

Thirdly, employment as an indicator is easier to measure and monitor when dealing with the extents of global competition, e.g. among the tech and educational hubs.

Which areas are the main competitors for businesses and jobs with Cambridge, globally?

The analysis carried out in this study suggests that the tech hubs that are competing most strongly with the Greater Cambridge city region are all outside the UK, due to the specific specialisms in research, development and education. The relatively large volumes of tech investments in the US also accentuate this effect, relative to other countries. This is also the conclusions reached by previous studies, such as CPIER (2018) and MHCLG (2024).

Among the UK tech centres, we observe exchange of jobs and workers and since it is assumed that the UK tech centres are subject to policy actions of the same government, they are likely to suffer from similar policy gaps or benefit from the same policy actions.

What are the mechanisms that cause employment to grow, decline and move?

The questions regarding employment location are very complex and there are many challenges in modelling (see Pagliara et al, 2013 for a comprehensive review). This study has approached the questions through modelling

(1) the employers (represented by the number of workplace jobs in each industry sector) – they can choose to increase, decrease or move jobs

(2) the employees (including self-employment, represented by employed residents in the city region) – they can choose where to live (and since covid, such choices also include remote locations outside the UK)

(3) the demand for local services (health, education, retail, hospitality, local government and other private services)

The modelling takes the CPIER central projection of employment growth in the city region as the baseline, and model respectively

(1) the employers – for existing exogenous industries (which are all except local-service industries), during each decade, 5% of the workplace jobs will look at the production costs, local investment pools available and the sizes of urban agglomeration to decide whether to move, and the other 95% will remain in the city region; for the new jobs that are expected in the CPIER projection, the workplace jobs will look at the production costs, local investment pools available and the sizes of urban agglomeration to decide whether to stay in the city region or move to another location;

(2) the employees – once the workplace jobs settle in their respective model zones, the employees' residential choices are modelled in a spatial equilibrium process, subject to costs of living, travel costs, and quality of life attractiveness. This includes local, wider UK and overseas locations (e.g. on fairly rare occasions, working remotely from an office outside the city region or in another country);

(3) the local services – once the employees settle in their respective model zones, the model estimates the demand for local services employment, i.e. the level of local services

employment will grow or decline in line with this demand, subject to the respective service catchments, etc.

The worrying declines in the level of employment as predicted by the model arises from

(a) strong post pandemic competition from the US tech centres in terms of stable production costs, the depth of investment pools, and the effects of much larger sizes of urban agglomeration;

(b) rise in Greater Cambridge city region's housing rents over time;

(c) the return of traffic congestion on the core weekdays for work;

(d) once the growth has lagged, the relative gap of the overall size of urban agglomeration effects, particularly in relation to the tech industry clusters would kick in, which causes the declines in the longer term along with the reduction in demand for the local services jobs.

Overcoming workplace employment data issues through modelling test options

There is one important caveat regarding the employment data used for 2021 in the model tests. As consistent with previous CF3 model runs, the employment data used in the model comes from the Census Workplace Population series (net of full time students who are working). The covid lockdown during the Census period has unfortunately made the 2021 data less comparable with the previous Census years, in that some employed residents who were not usually working from home may have reported in the Census forms as home-working, and thus artificially depressed the level of employment recorded in Census 2021 for some centres of employment.

Another Covid-related issue is to do with the uncertain extents of flexible working going forward. Flexible working has generally had the effect of reducing peak time traffic, although local data in the city region shows that the effects are uneven, and there is more reduction in public transport demand than in car traffic.

Table 3: Definition of model tests to mitigate employment data issues

	Employment data Use Census 2021 workplace population data directly	Use trend projection from the Joint Local Plan consultation for 2021
Flexible working patterns Current patterns persist over time	**Test (a)**	Test (c)
Reduced flexible working over the decade to 2031	Test (d)	**Test (b)**

To mitigate these effects four alternative tests are run with combinations of the above effects (see Table 1 below). These are combinations of the following two assumptions:

(1) Employment data: either using Census 2021 workplace population data directly or use a trend projection from the Joint Local Plan consultation that is based pre-covid estimation of

workplace employment for 2021. The latter option has a higher level of employment which implies higher housing and transport demands;

(2) Flexible working patterns: either assuming the current patterns persist over time or reduced flexible working over the decade to 2031 (the latter reflects the desire of many employers to bring workers back to offices, etc). The latter has a higher level of employment which implies higher housing and transport demands.

The definition of the tests (a) to (d) implies that (a) and (b) are expected to show the widest range of difference. This is because Test (a) represents a relatively low demand for housing and transport while Test (b) a relatively high demand. There are some nuances among the tests but they serve to show that regardless of the test options, the trends of employment decline are present and similar in all the tests.

Appendix A: The LUISA Model and Its Application to the Current Study

The core methodology of this study is based on a recursive spatial equilibrium theory for modelling the evolution of urban activities at a city region scale, as outlined in our research papers e.g. Jin et al (2013), and Echenique et al (2013). This builds on a tradition of more than 50 years at the Martin Centre of modelling the interactions among land use, built form, business and consumer activities and transport services (Echenique, 1967; 1994; forthcoming; UK Research Excellence Framework, 2014). One central feature of this Cambridge approach is its emphasis upon simultaneously solving the employment location model with the production, trade, residential location and transport demand models for any specific year (Pagliara et al, 2013, p6).

This model theory incorporates desirable features from:

(a) spatial computable general equilibrium modelling which provides a rigorous framework for predicting rents, wages and prices, and

(b) dynamic disequilibrium modelling which acknowledges the uncertain timing and indivisibility of many supply-side interventions and the unpredictability of many events in the wider economy.

The resulting recursive spatial equilibrium model is capable of predicting how businesses and individuals trade off job, housing and travel choices across a city region subject to explicit scenario assumptions regarding the timing and extent of supply-side interventions in both private and public sectors.

The new data sources such as observed wages, housing rents/prices and road congestion at the micro level have greatly extended the capability of this model in representing market equilibria. Census and business surveys (both nation-wide by ONS and local studies such as by CBR) now also provide fine-grained information on population and job locations.

The recursive spatial equilibrium theory is encapsulated in a MATLAB based software app that is documented as LUISA, at the Martin Centre. The specific version of the software app used for this study is LUISA2.11. For further details on the model structure and equations, see Appendix below. The implementation of the software app for this study is called the Cambridgeshire and Peterborough Futures Model.

This model appendix is organized as follows. Section A1 introduces the formal structure of the LUISA2.11 model which is a specially adapted version of the model that is used for this study. Section A2 discusses the model solving algorithm in a step-by-step manner. Section A3 summarizes the zoning system in the model. Lists of model variables and parameters are provided in Section A4.

A1 Structure of the LUISA2.11 Model

Suppose that the city region is divided into $\mathfrak{J}$ core zones plus $\wp$ peripheral zones. Core zones represent the core study area where detailed policy analyses are conducted with relatively fine spatial granularity; while the peripheral zones represent the wider region outside the core study area which exchanges production factors (e.g. labour) and trades goods & services with the core zones. $\mathbb{N} = \mathfrak{J} + \wp$ thus denotes all modelled zones, and specifically, $\wp$ includes an overseas competition zone. Each of the model zones has $r = 1, \ldots, \mathcal{R}$ basic industries and $f = 1, \ldots, F$ consumer types. Table A1.1 summarizes the model segmentations in the model.

TABLE A1.1 SEGMENTATIONS IN THE MODEL

	Industry types	Consumer types	Residential floorspace types	Commercial floorspace types
Core zones	$r = 1, \ldots, \mathcal{R}$	$f = 1, \ldots, F$	$m = 1, \ldots, \aleph_1$	$k = 1, \ldots, \aleph_2$
Peripheral zones	$r = 1, \ldots, \mathcal{R}$	$f = 1, \ldots, F$	$m = 1, \ldots, \aleph_1$	$k = 1, \ldots, \aleph_2$

We introduce the following model components in turn: producers, final consumers, location choices, stock constraints and equilibrium conditions.

A1.1 Producers

The producers, when endogenously modelled, are represented by a set of production functions that define how they use capital, labour, floorspace and intermediate inputs (raw materials and services). For this version of the model, only those producers of goods and services that the final consumers decide the quantities to purchase and the locations to source are modelled endogenously; the rest of the industries for which the production quantities and locations are not decided by the final consumers are represented directly as the number of people employed at each location. For endogenously modelled producers, a nested Cobb-Douglas CES (CD-CES) function has been broadly accepted as a standard for this purpose in spatial general equilibrium analyses since Krugman (1991) and Fujita et al. (1999). We follow Anas and Liu (2007) and Jin et al. (2013), and define the production function as a variant of the CD-CES specification.

$$X_{rj} = E_{rj} A_{rj} (K_r)^{\nu_r} \left(\sum\nolimits_f \kappa_{rfj} L_{fj}^{\theta_r}\right)^{\frac{\delta_r}{\theta_r}} \left(\sum\nolimits_k \chi_{rkj} B_{kj}^{\zeta_r}\right)^{\frac{\mu_r}{\zeta_r}} \prod\nolimits_s (Y_{rsj})^{\gamma_{rs}} \tag{1}$$

where X_{rj} is the production output of industry r in zone j; K_r, L_{fj}, B_{kj} and Y_{rsj} are the capital, labour, business floorspace and intermediate input, respectively; ν_r, δ_r, μ_r and γ_{rs} are cost share parameters for the respective input group. This function is Cobb-Douglas and is constant returns to scale by $\nu_r + \delta_r + \mu_r + \sum_s \gamma_{rs} = 1$. The elasticity of substitution between any two labour and building floorspace varieties is $1/(1-\theta_r)$ and $1/(1-\zeta_r)$, respectively. $\kappa_{rfj}, \chi_{rkj} \geq 0$ are input-specific constants for labour and business floorspace varieties, respectively. These constants allow us to specify input-specific preference within each input bundle. A_{rj} is a function of the economic mass for industry r in zone j that represents Hicksian-neutral Total Factor Productivity (TFP) effects resulting from learning and transfer of tacit knowledge (Graham & Kim, 2008; Rice, Venables, & Patacchini, 2006), which is an important component of urban agglomeration effects. E_{rj} is a constant scalar representing any additional zonal effects on total factor productivity. We define $A_{rj} = \underline{A}_{rj}(M_j/\underline{M}_j)^{\pi}$, where $\underline{A}_{rj}$ is a constant representing the baseline agglomeration effects, M_j is a function of the economic mass of zone j, $\underline{M_j}$ is a constant representing the baseline economic mass in j; π is a scale parameter. The function of economic mass builds on the concept of effective density (Graham, Gibbons, & Martin, 2009).

$$M_j = \sum_f \sum_i \frac{L_{fi}}{\chi_{fij}} \tag{2}$$

where L_{fi} is the total size of labour type f in zone i (including zone j) that is relevant to production zone j, and χ_{fij} is the travel time from location i to j for labour type f.

We assume that each firm minimizes the cost subject to the production demand and the price of each input variety. The *conditional input demand* (given target output X_{rj}) of each input factor can be derived as follows:

$$K_r = \frac{1}{\rho} \nu_r p_{rj} X_{rj} \tag{3}$$

$$L_{rfj} = \frac{\kappa_{rfj}^{\frac{1}{1-\theta_r}} w_{fj}^{\frac{1}{\theta_r-1}}}{\sum_s \kappa_{rsj}^{\frac{1}{1-\theta_r}} w_{sj}^{\frac{\theta_f}{\theta_r-1}}} \delta_r p_{rj} X_{rj} \tag{4}$$

$$B_{rkj} = \frac{\chi_{rkj}^{\frac{1}{1-\zeta_r}} R_{kj}^{\frac{1}{\zeta_r-1}}}{\sum_s \chi_{rsj}^{\frac{1}{1-\zeta_r}} R_{sj}^{\frac{\zeta_r}{\zeta_r-1}}} \mu_r p_{rj} X_{rj} \tag{5}$$

$$Y_{rsj} = \frac{\gamma_{rs} p_{rj} X_{rj}}{p_{rs|j}^{*}} \tag{6}$$

where p_{rj} is the unit production price of industry r in zone j; ρ is the exogenous price of business capital (i.e. the real interest rate); w_{fj} is the hourly wage of labour type f; R_{kj} is the average rent for business floorspace type k; and $p_{rs|j}^{*}$ is the average delivered price of intermediate input type s for producing product type r in zone j.

The minimized production price can then be calculated by substituting the above conditional demands into the production function. As zero profit is assumed at any level of output, the minimized price equals the average and the marginal cost, which takes the form:

$$p_{rj} = \frac{\rho^{\nu_r} \left(\sum_f \kappa_{rfj}^{\frac{1}{1-\theta_r}} w_{fj}^{\frac{\theta_r}{\theta_r-1}} \right)^{\frac{\delta_r \theta_r - 1}{\theta_r}} \left(\sum_k \chi_{rkj}^{\frac{1}{1-\zeta_r}} R_{kj}^{\frac{\zeta_r}{\zeta_r-1}} \right)^{\frac{\mu_r \zeta_r - 1}{\zeta_r}} \prod_m {p_{rs|j}^{*}}^{\gamma_{rs}}}{E_{rj} A_j \nu_r^{\nu_r} {\delta_r}^{\delta_r} \mu_r^{\mu_r} \prod_s \gamma_{rs}^{\gamma_{rs}}} \tag{7}$$

A1.2 Final Consumers

Final consumers are categorized into $f = 1, \dots, F$ types according to their employment status and socio-economic level. H_f is the exogenous number of consumers in group f. For city regional scale modelling, the final consumers include only residential population in this version of the model. Consumers in socio-economic group f receive both wage and nonwage income, except group $f = F$ denoting the non-employed consumers who do not have wage income but receive nonwage income through social welfare transfer. The wage income is modelled endogenously subject to equilibrium conditions, while the nonwage income is subject to the *a priori* welfare transfer scheme.

Each consumer makes a set of discrete and continuous choices. For discrete choices, the employed residents decide where to work and where to live jointly from $j = 1, \dots, \mathbb{N}$ employment zones and $i = 1, \dots, \mathbb{N}$ residence zones; the non-employed residents choose their residence location from $i = 1, \dots, \mathbb{N}$ residence zones. Both the employed and non-employed consumers choose where to source goods & services from $z = 1, \dots, \mathbb{N}$ production zones. The remaining choices entail continues variables and are conditional on the above discrete location choices. Consumers then decide on: 1) the annual consumption of each goods & services variety; 2) the quantity of type m housing floorspace to rent; 3) the use of time between work and leisure in the case of employed consumers. All consumers are assumed to maximize their utility from the mixed discrete-continuous choice.

Following the random utility framework (McFadden, 1973), the utility of consumer type f living in zone i and working in zone j takes the form $U^*_{fij} = U_{fij} + e_{fij}$ where U_{fij} is the observable quantity-based utility and e_{fij} is the error term which measures the unobservable utility variance among consumers. The observable utility U_{fij} is given by:

$$U_{fij} = \alpha_f \ln\left(\sum_r \sum_z \xi_{rfz}\left(Z_{rz|fij}\right)^{\eta_f}\right)^{\frac{1}{\eta_f}} + \beta_f \ln\left(\sum_m \iota_{mfi}\left(b_{m|fij}\right)^{\sigma_f}\right)^{\frac{1}{\sigma_f}} + \gamma_f \ln l_{fij} \tag{8}$$

$$\textit{subject to budget constraint}: \sum_{r,z}\left(p_{rz} + c_f 2g_{fiz}\right)Z_{rz|fij} + \sum_m r_{mi} b_{m|fij}$$
$$+ \Delta_f 2Dg_{fij} = \Delta_f w_{fj}\left(N - 2DG_{fij} - \sum_{r,z} c_f Z_{rz|fij} 2G_{fiz} - l_{fij}\right) + \mathcal{M}_{fi}$$

$$\textit{and time constraint}: \quad N - \sum_{r,z} c_f Z_{rz|fij} 2G_{fiz} - \Delta_f\left(l_{fij} + 2DG_{fij}\right) \geq 0$$

In equation (8), we assume Cobb-Douglas preference between goods & services $Z_{rz|fij}$, housing $b_{m|fij}$ and leisure time l_{fij}. $\alpha_f + \beta_f + \gamma_f = 1$ are the expenditure coefficients for each consumption bundle. The varieties of goods & services and housing are assumed to be imperfect substitutes (Dixit & Stiglitz, 1977), and the elasticity of substitution is governed by η_f and σ_f for goods & services and housing, respectively. $\xi_{rfz}, \iota_{mfi} > 0$ are the input-specific constants measuring the inherent attractiveness of the goods & services, and housing varieties for consumers type f, which is calibrated empirically.

For the budget constraint in equation (8), the right-hand side of the function is the total income and the left-hand side is the total expenditure. Specifically, p_{rz} is the mill price for goods & services type r produced in zone z; g_{fiz} and G_{fiz} is the expected one-way monetary cost and travel time from i to z for customers type f, respectively[4]; c_f is an exogenous coefficient that measures the cost for delivering a unit of goods & services as percentage of the normal trip cost. r_{mi} is the housing rent of type m in zone i; w_{fj} is the hourly wage rate for labour type f working in zone j. Δ_f is the employment status of the consumer type f. For all employed consumers $\Delta_f = 1$; otherwise $\Delta_f = 0$. $\mathcal{M}_{fi}$ is the nonwage income of consumer type f in zone i. It consists of normal investment returns on real estate in the city region (endogenous in the model) as well as the individual share of social welfare transfer and amenity gains (subject to *a priori* scheme). As for the time constraint, D is the exogenous number of working days per annum; $N = 24D$ is the exoge*n*ous total annual time endowment. For the non-employed consumers ($\Delta_f = 0$), the model only accounts for the time for shopping, as they do not commute and have zero value of time for leisure time.

We can rewrite the budget constraint in equation (8) to consider the value of time for shopping travel as a part of the delivered price. The new constraint function is equivalent to equation (8).

$$\sum_{r,z} p^*_{rz|fij} Z_{rz|fij} + \sum_m r_{mi} b_{m|fi} + \Delta_f 2D g_{ij} \tag{9}$$

$$= \Delta_f w_{fj}\left(N - 2DG_{ij} - l_{fij}\right) + \mathcal{M}_{fi}$$

where $p^*_{rz|fij}$ is the full delivered price of a unit of goods & services type r produced in zone z purchased by consumer type f living in zone i and working in zone j. We use the subscript z to denote the production location of goods & services and j as the employment location for employed workers. The full delivered price for final consumers $p^*_{rz|fij}$ is given by:

$$p^*_{rz|fij} = p_{rz} + c_f 2\left(g_{iz} + \Delta_f G_{iz} w_{fj}\right) \tag{10}$$

Accordingly, the full disposable income of the consumer type (fij) net of commuting costs is given by:

$$\Omega_{fij} = \Delta_f w_{fj}\left(N - 2DG_{ij} - l_{fij}\right) - \Delta_f 2D g_{ij} + \mathcal{M}_{fi} \tag{11}$$

Under the above budget and time constraint, we can then derive the *Marshallian* demand for goods & services, housing and leisure time in Eq. 3.12, Eq. 3.13 and Eq. 3.14, respectively.

[4] The monetary cost and travel time is composite over all available travel modes. For the moment, we do not consider the time-of-day and purpose variations in travel time and cost.

$$\bar{Z}_{r|fij} = \frac{\xi_{rfz}^{\frac{1}{1-\eta_f}} \bar{p}_{r|fij}^{\frac{1}{\eta_f - 1}}}{\sum_s \xi_{rfz}^{\frac{1}{1-\eta_f}} \bar{p}_{s|fij}^{\frac{\eta_f}{\eta_f - 1}}} \alpha_f \Omega_{fij} \tag{12}$$

$$b_{m|fij} = \frac{\iota_{mfi}^{\frac{1}{1-\sigma_f}} r_{mi}^{\frac{1}{\sigma_f - 1}}}{\sum_s \iota_{si}^{\frac{1}{1-\sigma_f}} r_{si}^{\frac{\sigma_f}{\sigma_f - 1}}} \beta_f \Omega_{fij} \tag{13}$$

$$l_{fij} = \frac{\gamma_f \Omega_{fij}}{w_{fj}} \tag{14}$$

where $\bar{Z}_{r|fij}$ is the aggregate demand for product type r for consumer type (fij); $\bar{p}_{r|fij}$ is the probability-weighted average price of product type r faced by consumer type (fij). The formulation of $\bar{p}_{r|fij}$ and $\bar{Z}_{r|fij}$ and the associated discrete-choice probability function will be introduced shortly.

In addition to the *Marshallian* utility function (maximizing utility subject to budget constraints), which is used in base-year model calibration, the model employs the *Hicksian* utility function in forecasts. The Hicksian utility function differs from the Marshallian utility function in that it minimizes the expenditure given fixed utility. The use of Hicksian utility function in forecast mode implies that consumers are assumed to maintain, if not increase, their base-year utility level in future years by altering their locational and consumption choices. Under the same Nested-CES configuration and parameterization, the Marshallian and Hicksian utility functions are consistent in base-year model calibration, in the sense that the derived Marshallian demands (given observed budget constraint) are identical to the Hicksian demands (given the Marshallian utility). In forecast mode, the Hicksian utility function will replace the Marshallian utility function. The implication is that consumers will have to raise the income if the cost of living (i.e. prices of goods & services and housing rents) goes up, in order to maintain the same utility level. The need for increasing income will then be represented by an upward pressure on labour wage. In case the cost of living goes down (e.g. abundance of housing supply), the model assumes that the local wage level would not decrease subject to global price adjustment. Nonetheless the resulting extra utility gain will be competed out in spatial equilibrium as more residents move into the area, which in turn drives up the cost of living. For the Hicksian utility function, the minimized expenditure given the utility U_{fij} is defined as:

$$\Omega_{fij}^{Hicksian} = \alpha_f^{-\alpha_f}\beta_f^{-\beta_f}\gamma_f^{-\gamma_f}\left[\left(\sum_r\sum_z \xi_{rfz}^{\frac{1}{1-\eta_f}}\bar{p}_{r|fij}^{\frac{\eta_f}{\eta_f-1}}\right)^{\frac{\eta_{f-1}}{\eta_f}}\right]^{\alpha_f}$$

$$\left[\left(\sum_m \iota_{mfi}^{\frac{1}{1-\sigma_f}} r_{mi}^{\frac{\sigma_f}{\sigma_f-1}}\right)^{\frac{\sigma_f-1}{\sigma_f}}\right]^{\beta_f} (w_{fj})^{\gamma_f} U_{fij} \tag{15}$$

The total annual labour working time N_{fij} for the employed consumer type (fij) is thus determined by subtracting the total travel time for commuting and shopping, and the annual leisure time from the annual time endowment N.

$$N_{fij} = N - 2DG_{ij} - \sum_{r,z} c_f Z_{rz|fij} 2G_{iz} - l_{fij} \geq 0 \tag{16}$$

The next step is to evaluate the direct utility function (8) to get the price-based indirect utility function $\widetilde{U}_{fij}$, which is given by:

$$\widetilde{U}_{fij} = \ln\Omega_{fij} - \alpha_f\frac{\eta_f - 1}{\eta_f}\ln\left(\sum_r\sum_z \xi_{rfz}^{\frac{1}{1-\eta_f}}\bar{p}_{r|fij}^{\frac{\eta_f}{\eta_f-1}}\right)$$

$$- \beta_f\frac{\sigma_f - 1}{\sigma_f}\ln\left(\sum_m \iota_{mfi}^{\frac{1}{1-\sigma_f}} r_{mi}^{\frac{\sigma_f}{\sigma_f-1}}\right) - \gamma_f \ln w_{fj} \tag{17}$$

Note that the quantity-based and the price-based utility functions are mathematically equivalent in static equilibrium. However, for the purpose of welfare evaluation over time, particularly in long-term forecast that involves macroeconomic changes (e.g. price-level changes due to growth, inflation or deflation), the quantity-based direct utility function offers a more intuitive and straightforward measure than the price-based counterpart. Therefore, we use the price-based utility in static equilibria and the quantity-based utility for welfare analysis.

A1.3 Location Choices

The location choices in the model include: (1) endogenous goods & services whose locations are directly determined by final consumers in the UK; (2) exogenous industries whose locations are not directly determined by final consumers in the UK, such as those of intermediate products and exporting industries; (3) the employment-residence choices (or residence location choices only if the employment location is exogenous) for the employed persons. Both location choices (1) and (3) are modelled in the spatial equilibrium framework where location choices are influenced by costs of living (including housing costs) and travel disutility, whilst (2) follows an assumed baseline pattern of distribution that is modified by relative changes over time in each scenario in terms of production prices and agglomeration effects. We summarize the respective models below.

A1.3.1 Endogenous goods and services for final consumers in the UK

For endogenous goods and services, the final consumers decide both the quantity of each product to purchase and where to source them. The former decision is based on average delivered price of each product thus is continuous in nature; while the latter choice is discrete involving location alternatives. We represent this mixed discrete-continuous choice problem by combining two different choice models. For the continuous choice on quantities, a nested CES function is applied to consider the substitution effects within the consumption bundle (see Section above). For the discrete location choice, the sourcing pattern is modelled with a multinomial logit probabilistic model. The probability of obtaining product type r from zone z to consumer type f living in zone i (and working in zone j, if employed) is given by:

$$P_{rz|fij} = \frac{S_z \exp\left(-\lambda_{f|r}(p_{rz} + c_f \chi_{fiz} + \psi_{riz} - E_{rfz})\right)}{\sum_n S_n \exp\left(-\lambda_{f|r}(p_{rn} + c_f \chi_{fin} + \psi_{rin} - E_{rfn})\right)} \quad (18)$$

where S_z is a size term that corrects for the bias introduced by the uneven sizes of zones in the model (Ben-Akiva & Lerman, 1985); $\lambda_{f|r}$ is the dispersion parameter. c_f is a coefficient measuring the cost for delivering a unit of goods or services as percentage of normal trip cost; χ_{fiz} is a travel disutility function; ψ_{riz} are observable non-monetary barriers for trading between zone i and zone z; E_{rfz} is the residual attractiveness term which is calibrated empirically. In the model, consumers will shop to all potential production zones, rather than the zone with the cheapest delivered price only[5]. A similar probability function can be applied to model the sourcing of intermediate inputs for producers.

With the above probability, we can derive the weighted average price of product type r faced by consumer type (fij). Note that this weighted average price considers the consumption inputs from all possible production locations, thus the dimension is $[r]$.

$$\bar{p}_{r|fij} = \sum_z p^*_{rz|fij} P_{rz|fij} \quad (19)$$

where $p^*_{rz|fij}$ is the full delivered price including the value of time for travel. The purpose of deriving $\bar{p}_{r|fij}$ is to link the discrete location choice with the continuous choice of consumption quantities. For residents living in zone i, they first choose how much to consume for each product type ($\bar{Z}_{r|fij}$), regardless of the their production locations. This continuous choice is made based on the weighted average price $\bar{p}_{r|fij}$ through CES functions. The discrete-choice probability in Eq. 3.17 then distributes the aggregate demand $\bar{Z}_{r|fij}$ to each production location z. This distribution process is given by:

$$Z_{rz|fij} = P_{rz|fij} \bar{Z}_{r|fij} \quad (20)$$

This function is used to derive the total production demand for product type r in zone z. This total production demand is then used to derive the total labour and business floorspace demands at each workplace location j.

[5] By "shop" we refer to any non-work trip that involves the purchase of goods and services. We approximate trip chains and travels that do not originate from home through home-shop trips where physical trips are involved.

A1.3.2 Exogenous industries

Exogenous industries follow an assumed baseline pattern of distribution that is modified by relative changes over time by scenario, in terms of production prices, size of available local investment pool and of agglomeration effects.

These assumptions are naturally contingent upon the contexts of the case study area as well as purposes of the study. For this study, the overall baseline growth/decline in employment in the exogenous industries is first worked out using the UK's projected growth in employment and per work productivity, net of the endogenous industries (see A1.3.1 above). Then for the Greater Cambridge area, the employment growth assumptions follow those from the on-going Local Plan Consultation. For details see Appendix B below.

This overall growth/decline is then distributed to the model zone level through a stayer-mover model as per Jin et al (2013), in which the employment of industry r in model zone j in period (t+1), or Λ_{rj}^{t+1}, is computed as

$$\Lambda_{rj}^{t+1} = \bar{\Lambda}_{rj}^{t} + \tilde{\Lambda}_{rj}^{t} + \Delta\bar{\Lambda}_{rj}^{t+1} + \Delta\tilde{\Lambda}_{rj}^{t+1} \qquad (21)$$

where the four components on the right of the equation are defined as follows.

The employment that stays on in zone j, $\bar{\Lambda}_{rj}^{t}$ is a proportion, η_{rj}^{t+1}, of the total employment of industry r in zone j:

$$\bar{\Lambda}_{rj}^{t} = \eta_{rj}^{t+1}\Lambda_{rj}^{t} \qquad (22)$$

The employment that moves at the end of period t is a share of all those movers that are predicted by the following discrete choice model

$$\tilde{\Lambda}_{rj}^{t} = \frac{S_{rj}^{t+1} exp\,(\lambda_r^{t+1} W_{rj}^{t+1})}{\sum_k S_{rk}^{t+1} exp\,(\lambda_r^{t+1} W_{rk}^{t+1})} \sum_j (1 - \eta_{rj}^{t+1}) \Lambda_{rj}^{t} \qquad (23)$$

where S_{rj}^{t+1} is a size term to correct the effects arising from uneven zone sizes for sector r in zone j and period (t+1), W_{rj}^{t+1} is the attractivity weight for employment in sector r, zone j and period (t+1) that is defined by production prices, size of available local investment pool and agglomeration effects. λ_r^{t+1} is a model parameter. Note that the choice model should apply to only a subgroup of zones j (including external zones) where there is relevant competition among the zones for employment in sector r.

The employment increment in sector r from period t to (t+1) that is dedicated to zone r is modelled by:

$$\Delta\bar{\Lambda}_{rj}^{t+1} = \hat{\eta}_{rj}^{t+1}(\Delta\Lambda_{rj}^{t}) \qquad (24)$$

Whereas the employment increment in sector r from period t to (t+1) that is free to choose among a number of competing zones r is modelled by the following discrete choice model

$$\Delta\tilde{\Lambda}_{rj}^{t} = \frac{\hat{S}_{rj}^{t+1} exp\,(\hat{\lambda}_r^{t+1} \hat{W}_{rj}^{t+1})}{\sum_k \hat{S}_{rk}^{t+1} exp\,(\hat{\lambda}_r^{t+1} \hat{W}_{rk}^{t+1})} \sum_j (1 - \hat{\eta}_{rj}^{t+1})(\Delta\Lambda_{rj}^{t}) \qquad (25)$$

where $\hat{S}_{rj}^{t+1}$ is a size term to correct the effects arising from uneven zone sizes for sector r in zone j and period (t+1), $\widehat{W}_{rj}^{t+1}$ is the attractivity weight for employment in sector r, zone j and period (t+1) that is defined by production prices, size of available local investment pool and agglomeration effects. $\hat{\lambda}_{r}^{t+1}$ is a model parameter. Similarly, note that the choice model should apply to only a subgroup of zones j (including external zones) where there is relevant competition among the zones for employment in sector r.

A1.3.3 Employment/residence location choice

In the model, we differentiate the location choice of employed residents and the non-employed. For employed residents we assume that they respond quickly to the utility changes and are mobile in terms of employment-residence relocation in static equilibria. By contrast, the relocation of non-employed residents is inertia-prone, i.e. there may be a lag of many years between a utility change and household relocation. We thus deal the relocation of non-employed households outside the equilibrium framework through recursive dynamic model or model assumptions. This section first introduces the discrete choice model for employment-residence joint choice. The residence location choice model as an abridged version the former model will be discussed afterwards.

For the employment-residence choice of employed residents, a multinomial logit model is developed. The probability of consumer f working in zone j choosing to live in zone i is defined as:

$$P_{fij} = \frac{S_{ij}\exp\left(\lambda_f v_{fij}\right)}{\sum_{m,n} S_{mn}\exp\left(\lambda_{fI} v_{fmn}\right)} \tag{26}$$

where

$$v_{fij} = \widetilde{U}_{fij} - d_{fij} + \psi_{fij} + E_{fij} + e_{fij} \tag{27}$$

S_{ij} is the a size term that addresses the size of residence/employment opportunities in zone i/j; $\lambda_{f|I}$ is the dispersion parameter; $\widetilde{U}_{fij}$ is the consumption utility of consumer f living in zone i and working in zone j; d_{fij} is the travel disutility of travelling from zone i to j; E_{fij} is the residual attractiveness of location pair (i,j), and e_{fij} is the unobserved error term.

For the residence choice of employed residents, the probability of consumer f choosing to live in zone i, given the employment location j, is defined as:

$$P_{fi|j} = \frac{S_i\exp\left(\lambda_{f|I} v_{fi|j}\right)}{\sum_m S_m\exp\left(\lambda_{f|I} v_{fm|j}\right)} \tag{28}$$

where

$$v_{fi|j} = \widetilde{U}_{fi|j} - d_{fi|j} + \psi_{fi|j} + E_{fi|j} + e_{fi|j} \tag{29}$$

$v_{fi|j}$ is the residence location utility of zone i for resident type f, given the chosen workplace j; $\lambda_{f|I}$ is the dispersion parameter. The other variables follow the same definitions as in function v_{fij}, except that the employment location j is given.

A1.3.4 Travel disutility

In the model, the χ_{fij} function is introduced to represent the attributes of travel for traveller type f from i to j. We differentiate the χ_{fij} function for different uses throughout the model. In this section, we summarize the use of the χ_{fij} function. For measuring the economic mass (as in Eq. 2), we define $\chi_{fij} = 2G_{fiz}$, which is the round-trip travel time (in hourly term) between zone i and j for traveller type f.

For sourcing goods & services (as in Eq. 18), we define $\chi_{fiz} = 2(g_{fiz}/\varsigma_f \bar{w}_{fi} + G_{fiz})$, where $\bar{w}_{fi}$ is the average hourly wage of type-f employed residents living in zone i[6], and $\varsigma_f \in (0,1]$ is a decay coefficient, implying that the shopping trip being partly voluntary thus its value of time is not fully valued by the traveller. The front multiplier transforms the one-way cost into round-trip cost (de Dios OrtÃozar & Willumsen, 2011). The above formulation adopts the time unit (hour), and considers both the travel time and the monetary cost. The monetary cost is transformed into time unit by dividing it by the value of time $\varsigma_f \bar{w}_{fi}$. Note that this time-based travel disutility is only used for modelling location choices. The actual transport costs, including the value of time, are measured in monetary unit in the equilibrating process.

For the employment-residence location choice, it is important to consider the realistic commuting patterns within a large city region. City regions with reasonably self-contained commuting catchment today tend to have a radius of 50km or more. At this metropolitan scale, extensive analyses of travel choices data show that a d_{ij} function (as in Eq. 22) that is linear to travel costs and times will have great difficulties in representing realistic demand elasticity throughout (Jin et al., 2013); a non-linear transformation of utilities is required (Gaudry & Laferriére, 1989). Fox et al (2009) devise a log-linear transformation that is a close equivalent to the Box-Cox function whilst being easier to calibrate. This function is given by:

$$d_{fij} = a_{f|d}\chi_{fij} + (1 - a_{f|d}) \ln \chi_{fij} - a_{f|d} \tag{30}$$

where $\chi_{fij} = 2DG_{fiz}$, i.e. the annual total commuting time between zone i and j for labour type f, and $a_{f|d}$ is a log-linear parameter. The reason why we do not account for the monetary cost is that the monetary cost is already accounted for in the consumption utility function (see the budget constraint in Eq. 8). To avoid double counting, we thus only consider the travel time in the χ_{fij} function.

Note that the modelled elasticity of the log-linear function varies for different distance ranges. Specifically, the elasticity of disutility with regard to distance is higher for short-distance range (approx. 0-15 km), and becomes lower for long-distance range (approx. > 15 km).

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[6] To distinguish $\bar{w}_{fi}$ and w_{fj}, the latter is the hourly wage of labour type f at production zone j, while the former is the average wage for labour type-f living in residence zone i, weighted by the modelled labour distribution to all employment locations.

A1.4 Stock Constraints

We define stock constraints to cover land/floorspace and transport infrastructure which may evolve or "churn" slowly. In the model, the stock constraints include: 1) the zonal supply of housing floorspace varieties ($\hat{b}_{mi}$) and business floorspace varieties ($\hat{B}_{ki}$); 2) the expected transport monetary cost (g_{fij}) and travel time (G_{fij}) for consumer type f; 3) the zonal number of non-employed residents (H_F).

In the model, such stock constraints remain exogenous for any static period and will be updated periodically in a non-equilibrium manner. The underlying assumption is that land/floorspace and transport infrastructure respond to demand slowly and indivisibly, subject to regulation, planning, construction, commission and decommission (Jin et al., 2013). User-defined supply scenarios are likely to be the most appropriate in order to reflect policy targets and background changes. As for the relocation of non-employed residents, it is assumed that there is a time lag between a utility change and household relocation.

A1.5 Equilibrium Conditions

The general equilibrium structure of the model requires three sets of equilibrium conditions to be satisfied simultaneously, conditional on the transport conditions $\boldsymbol{g}$ and $\boldsymbol{G}$.

1) All consumers maximize utility subject to budget and time constraint, or minimise expenditure subject to given utility target.
2) All producers minimize cost subject to supply constraint of input factors and technology. Producers are competitive and operate under constant returns to scale. The minimized production price equals the average and marginal cost, implying zero economic profit.
3) All markets clear with zero excess demands. This applies to: a) the residential and business floorspace markets; b) the labour market for each socio-economic group at each production zone; c) the product market of each product type at each production zone.

The above equilibrium conditions are formulated in the model as follows:

A1.5.1 Product markets

The market clearance condition in both zonal and regional product markets prescribes that in each of the $j = 1, \dots, \mathbb{N}$ production zone, the production output of each industry should equal the total production demand plus net export. Let $Y_{rj|sn}$ be the intermediate demand for industry r in zone j for producing product s in zone n and Ξ_{rj} be the exogenous net export for industry r in zone j. The zero excess demands in product markets require:

$$\sum_{f,z} H_{fi} P_{rz|fij} \bar{z}_{r|fij} + \sum_{s,n} Y_{rj|sn} + \Xi_{rj} = X_{rj} \tag{31}$$

A1.5.2 Labour Markets

In each of the $j = 1, \dots, \mathbb{N}$ production zone, the annual labour demand in hourly term for each of the $f = 1, \dots, F-1$ labour group must equal the working hours supplied by the respective labour group, net of the time for commuting, shopping and leisure.

$$\sum_{r} L_{rfj} = \sum_{i} H_{fi} P_{fij} \left(N - 2DG_{fij} - \sum_{r,z} c_f Z_{rz|fij} 2G_{fiz} - l_{fij} \right) \tag{32}$$

A1.5.3 Floorspace Markets

We treat the zonal building floorspace as exogenous supply constraints in static equilibria, and update them through Recursive Dynamic models. The market clearance in floorspace markets requires that in static equilibrium, the zonal demand for each type of residential and business floorspace must equal the corresponding zonal supply constraint.

$$\sum_{f,j} b_{m|fij} = \hat{b}_{mi} \tag{33}$$

$$\sum_{r} B_{rkj} = \hat{B}_{kj} \tag{34}$$

where $\hat{b}_{mi}$ and $\hat{B}_{kj}$ is the zonal supply constraint for housing and business floorspace, respectively.

As a summary, the aforementioned equilibrium conditions define the aggregate behavioural rules of agents, and specify how they interact with each other in respective market. In fact, the equilibrium conditions constitute the economic foundation of general equilibrium models, and it is a theoretical necessity to satisfy such conditions in equilibrium analysis.

A2 Model Algorithm

In the previous section, we present the formal structure of the Spatial Equilibrium model. Given the exogenous stock constraints (building floorspace supply, transport infrastructure and non-employed households), the aforementioned equations and variables complete the spatial general equilibrium of the model. Following the convention of spatial equilibrium models, we solve the static equilibrium in a sequential manner, which is specified in Figure 1.

The solving algorithm for the Spatial Equilibrium model is as follows:

STEP 0 (Initialization). Arbitrary exogenous vectors of rents $(\boldsymbol{R}, \boldsymbol{r})$, wages $(\boldsymbol{w})$ serve as initial inputs. Given the guessed values, as well as the given transport conditions $\boldsymbol{G}$ and $\boldsymbol{g}$ and all parameters, the following sequentially arranged steps complete a single iteration of the SE model.

STEP 1 (Production prices). The zero economic profit equation (7) is solved for the equilibrium production price $\boldsymbol{p}$, given wages $\mathbf{w}$ and business floorspace rents $\mathbf{R}$.

STEP 2 (Location choices). Residents make discrete location choice for sourcing goods & services with equation (18). Employed residents make joint location choices with Equation 21 or 23.

STEP 3 (Outputs). Given the production price $\boldsymbol{p}$ from STEP 1 and the location choices from STEP 2, the final demand for production $\boldsymbol{F}$ can be solved with the *Marshallian* demand function (12) and the zero-excess-demand equation (31). The total production demand $\boldsymbol{X}$, including the intermediate demand, can be derived with the classical input-output solution $\boldsymbol{X} = (\boldsymbol{I} - \boldsymbol{A})^{-1}\boldsymbol{F}$, where $\boldsymbol{A} = [\gamma_{rs}]$ is the matrix of input-output coefficients.

STEP 4 (Rents). Given the production price $\boldsymbol{p}$ from STEP 1 and the production outputs $\boldsymbol{X}$ from STEP 3, the equilibrium rents for business floorspace $\mathbf{R}$ can be solved with the floorspace demand function (3.5) subject to the stock constraints $\widehat{\boldsymbol{B}}$. Similarly, the housing rents $\mathbf{r}$ are solved with the *Marshallian* or *Hicksian* demand function subject to the housing stock constraints $\widehat{\boldsymbol{b}}$.

STEP 5 (Wages). Given the production price $\boldsymbol{p}$ from STEP 1, the location choices from STEP 2, and the production outputs $\boldsymbol{X}$ from STEP 3, the equilibrium wages $\mathbf{w}$ can be solved with the labour market zero-excess-demand equation.

STEP 6 (Updating). Gathering the results of STEP 1 to STEP 5, the algorithm has determined vectors $\boldsymbol{p}, \boldsymbol{w}, \boldsymbol{R}, \boldsymbol{r}$ conditional on transport matrices $\boldsymbol{G}$ and $\boldsymbol{g}$ and all exogenous variables, constraints and parameters. The algorithm will then check whether these updated prices and the associated quantities are converged and whether they simultaneously satisfy all equilibrium conditions to a desired level of accuracy that is discussed below. If not, then the next iteration is started by returning to STEP 1 with these updated vectors. If all equilibrium conditions and converging criteria are satisfied simultaneously, model iteration stops and writes output files.

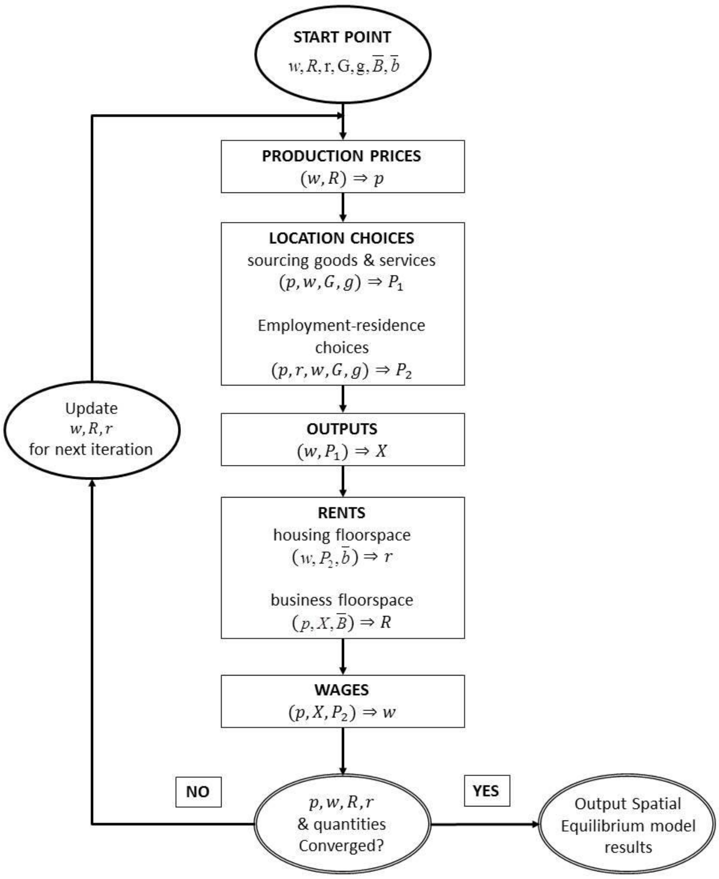

FIGURE 1 SOLVING ALGORITHM FOR SPATIAL EQUILIBRIUM MODEL

We define the level of converging accuracy by setting a maximum relative error condition. The Spatial Equilibrium model is considered converged in the nth iteration when the following inequality condition is satisfied simultaneously for all prices and quantities concerned:

$$\max_{\forall i}\left(\left|\frac{x_{i|n}-x_{i|n-1}}{\frac{1}{2}\left(x_{i|n}+x_{i|n-1}\right)}\right|\right) < ITERTOL \tag{35}$$

where vectors $x_{i|n}$ include zonal prices $\boldsymbol{p}, \boldsymbol{w}, \boldsymbol{R}, \boldsymbol{r}$ and all the associated excess demands in iteration n, and $ITERTOL$ is a user-specified maximum iteration tolerance. When the Spatial Equilibrium model is initiated with guesstimated starting values, large relative errors between iterations may occur. As the model approaches the equilibrium solution, the relative errors are expected to reduce gradually, yet not necessarily monotonically.

In order to stabilize the equilibrating process and avoid the model from divergence, we need to define how the variables are updated between iterations. Let $Current(X_n)$ be the variable value in iteration n and $New(X_{n+1})$ be the updated value from the solving algorithm for iteration $n+1$, we set:

$$Current(x_{n+1}) = \varpi(n)New(x_n) + [1-\varpi(n)]Current(x_n) \tag{36}$$

where coefficient $\varpi(n) \in [0,1]$ is a monotonically increasing function with respect to the iteration number $n \in [1, MAXITER]$. The $\varpi(n)$ function represents a smoothing technique for updating variables between iterations. A smaller step change of $\varpi(n)$ helps to stabilize the equilibrating process but incurs more iterations.

List of Variables in the Model

INDICES FOR DIMENSIONS OF THE MODEL	
$\mathfrak{I}$	Number of core zones
$\wp$	Number of peripheral zones
$\mathbb{N} = \mathfrak{I} + \wp$	Total number of model zones
F	Number of social-economic groups
$\mathcal{R}$	Number of industry types
$\aleph_1$	Number of residential floorspace types
$\aleph_2$	Number of business floorspace types
D	Exogenous number of annual working days
$N = 24D$	Exoge*n*ous total annual time endowment
VARIABLES IN SPATIAL EQUILIBRIUM MODEL	
X_{rj}	Aggregate production output of industry r in zone *j*
E_{rj}	Constant scalar representing any additional zonal effects on Total Factor Productivity (TFP)
A_{rj}	An economic mass function for industry r in zone j that represents the agglomeration effects on TFP
K_r	Capital input for industry r
L_{fj}	Endogenous labour input of type f for industry r in zone *j*
Λ_{fj}	Exogenous labour input of type f for industry r in zone *j*
B_{kj}	Business floorspace input of type k for industry r in zone *j*
Y_{rsj}	Intermediate input of type s for industry r in zone *j*
M_j	Economic mass of zone *j*
S_j	Geographic area or size term of zone *j*
χ_{fij}	Travel disutility function for socio-economic group type f travelling from i to *j*
p_{rj}	Unit production price of industry r in zone *j*
ρ	Real interest rate
w_{fj}	Hourly wage of labour type f in zone *j*
R_{kj}	Average rent for business floorspace type k in zone *j*
$p^*_{rs\|j}$	Average delivered price of intermediate input type s for producing product type r in zone *j*
U_{fij}	Observable utility of resident type f living in zone i and working in zone *j*
W_j	Attractivity weight of zone j
$Z_{rz\|fij}$	Aggregate consumption volume for industry r in zone z, given resident type f living in zone i and working in zone *j*
$b_{m\|fij}$	Consumption volume for housing type m in zone i, given resident type f living in zone i and working in zone *j*
l_{fij}	Leisure time of resident type f living in zone i and working in zone *j*

g_{fiz}	Expected one-way monetary cost from i to z for customers type f
G_{fiz}	Expected one-way travel time from i to z for customers type f
$\mathcal{M}_{fi}$	Nonwage income of consumer type f in zone i
r_{mi}	Housing rent of type m in zone i
Δ_f	Employment status of the consumer type f (For all employed consumers $\Delta_f = 1$; otherwise $\Delta_f = 0$)
$p^*_{rz\|fij}$	Full delivered price of a unit of goods & services type r produced in zone z purchased by consumer type f living in zone i and working in zone j
Ω_{fij}	Full disposable income of the consumer type (fij) net of commuting costs
$\bar{Z}_{r\|fij}$	Aggregate demand for product type r for consumer type (fij)
$\bar{p}_{r\|fij}$	Probability-weighted average price of product type r faced by consumer type (fij)
N_{fij}	Total annual labour working time for labour type (fij)
$\widetilde{U}_{fij}$	Price-based indirect utility of resident type f living in zone i and working in zone j
$P_{rz\|fij}$	Probability of obtaining product type r from zone z to consumer type f living in zone i (and working in zone j, if employed)
S_z	Size term that corrects for the bias introduced by the uneven sizes of zones in the model
P_{fij}	Probability of employed resident type f choosing to live in zone i and work in zone j
v_{fj}	Employment location utility of zone j for labour type f
$v_{fi\|j}$	Residence location utility of zone i for resident type f, given the chosen workplace j
$V_{f\|j}$	*Log-sum* or *inclusive utility* representing the expected utility that employed worker type f in zone j would receive from all residence location choices
$\overline{w}_{fi}$	Average hourly wage of type-f employed residents living in zone i
d_{fij}	Travel disutility after Box-Cox transformation for commuter type f travelling from i to j
$\hat{b}_{mi}$	Stock constraints of housing floorspace type m in zone i
$\hat{B}_{ki}$	Stock constraints of business floorspace type k in zone j
H_{fi}	Number of type f residents in zone i
Θ	Exogenous nonwage income from other sources
Ξ_{rj}	Exogenous net export for industry r in zone j
VARIABLES IN RECURSIVE DYNAMIC MODELS	
$\hat{B}^{t+1}_{ki}$	Zonal business floorspace stock of type k at zone i for period $t+1$
$\vec{B}^{t\|t+1}_{k}$	Regional aggregate stock change of business floorspace type k from period t to $t+1$
$V_{i\|B}$	Locational utility of zone j for business floorspace growth

$\hat{b}_{mi}^{t+1}$	Zonal housing floorspace stock of type m at zone i for period $t+1$
$\vec{b}_{m}^{t\|t+1}$	Regional aggregate stock change of housing floorspace type m from period t to $t+1$
$V_{i\|b}$	Locational utility of zone j for housing floorspace growth
$\bar{R}_i^t$	Zonal average business floorspace rent at zone i for period t
$\bar{R}_D^t$	Municipal/provincial average business floorspace rents at D for period t
$\mathcal{D}_i^t$	Zonal building floorspace density at zone i for period t
$\beth_{\mathrm{i\|B}}$	Dummy variable indicating zonal policy trend for business floorspace growth
$\bar{r}_i^t$	Zonal average housing floorspace rent at zone i for period t
$\bar{r}_D^t$	Municipal/provincial average housing floorspace rents at D for period t
$\beth_{\mathrm{i\|b}}$	Dummy variable indicating zonal positive policy trend for housing floorspace growth
$\gimel_{\mathrm{i\|b}}$	Dummy variable indicating zonal negative policy trend for housing floorspace growth
$H_{i\|F}^{t+1}$	Zonal number of non-employed residents in zone i at period $t+1$
$\vec{H}_F^{t\|t+1}$	Regional aggregate change of non-employed households from period t to $t+1$
J_{fj}^t	Number of labour type f in zone j for period t

List of Parameters in the Model

PARAMETERS IN SPATIAL EQUILIBRIUM MODEL	
δ_r	Labour cost share
μ_r	Business floorspace cost share
ν_r	Capital cost share
γ_{rn}	Intermediate cost share
ζ_r	Elasticity of substitution for business floorspace varieties
θ_r	Elasticity of substitution for labour varieties
σ_f	Elasticity of substitution for housing varieties
$a_{f\|\kappa}$	Coefficient for determining the input-specific parameters for labour varieties
κ_{rfj}	Input-specific parameters for labour varieties
$a_{f\|\iota}$	Coefficient for determining the input-specific parameters for housing varieties
ξ_{rfz}	Input-specific parameters for goods & services varieties
ι_{mfi}	Input-specific parameters for housing varieties
E_j	Additional total factor productivity multiplier
η_{rj}	Proportional parameter for sector r and zone j
π	Economic mass effects on productivity
c_f	Cost for delivering a unit of local services as percentage of commuting trip cost
α_f	Utility coefficient for goods & services
β_f	Utility coefficient for housing
γ_f	Utility coefficient for leisure time
$a_{f\|d}$	Log-linear travel cost function parameter
ς_f	Decay coefficient for value of time (non-commuting travels)
$\lambda_{f\|r}$	Dispersion parameter for sourcing goods & services
$\lambda_{f\|J}$	Dispersion parameter for employment location choices
$\lambda_{f\|I}$	Dispersion parameter for residence location choices
$\psi_{iz}, \psi_{fi\|j}, \psi_{fj}$	Observable non-monetary barriers for spatial interaction
E_{fz}	Residual attractiveness for sourcing goods & services
E_{fj}, $E_{fi\|j}$	Residual attractiveness for residence-employment location choices

Summary of the geographic zones of the model

Local Authorities in GCGP LEP	Number of Zones
Cambridge	13
South Cambridgeshire	20
East Cambridgeshire	10
Huntingdonshire	22
Fenland	11
Peterborough	22
Forest Heath	7
St Edmundsbury	14
North Hertfordshire	15
Uttlesford	9
King's Lynn and West Norfolk	19
Rutland	5
South Holland	11
South Kesteven	16
CPCA Area	**98**
GCGP LEP Area	**194**
Rest of the UK	**69**
Outside the UK	**2**
Total	**265**

Appendix C : Bibliography

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